

# **Introduction into the theory of vintage models**

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## Abstract:

This paper provides a general conceptual, theoretical framework for vintage growth models. These models allow for heterogeneous factor(s) of production. Usually labour is considered to be homogeneous and capital heterogeneous. The diversity of capital might be due to embodied technical change or can result from the difference between *ex ante* and *ex post* substitutabilities of the production factors. Depending on the sets of *ex ante* and *ex post* efficient activities vintage models can be basically of three types: *clay-clay*, *putty-clay* and *putty-putty*.

# Introduction into the theory of vintage models<sup>1</sup>

László Kónya

As far as our topic is concerned there are three common deficiencies of the traditional growth models based upon different aggregate production functions.

First, they pass over the age composition and life span structure of the stock of installed equipment. Second, they either simply leave technical progress out of consideration, or consider only the disembodied type. And finally, they assume that the *ex ante* and *ex post* substitutabilities between factors of production are identical.

As a result, in traditional growth models both labour and capital<sup>2</sup> are considered homogeneous factors of production, implying that if labour input is held fixed, the same amount of output can be produced with each unit of capital.

However, if we are interested in the age composition of the stock of installed equipment and want to trace its development - because we postulate, for example, that embodied technical progress (embodied in capital) is not a negligible source of economic growth, and/or assume that the *ex ante* and *ex post* substitutabilities of factors of production are different -, then we have to give up the myth of homogeneous capital and can not enjoy the advantage of such simplicity in modelling.

In this case it is already essential to group the elements of the capital stock according to their age, technical standard and feasible activity(ies), and to use vintage models, which relax releasing the restrictions mentioned above, and are superior to traditional growth models.

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<sup>1</sup> An earlier version of this paper was published in Hungarian as the first half of Kónya [1992].

<sup>2</sup> I use the word capital as a synonym for plant, machinery or equipment.

This is discussed in Sections 1 and 2 of this paper. Section 3 focuses on the concept, while Section 4 on the classification of vintage models.

## 1. Embodied Technical Progress

Owing to the appearance and application of new scientific accomplishments, to the development of new manufacturing processes and to the improving organisation of production the set of feasible activities of a given economic unit is broadening over time. The qualitative aspect of this course, however, can only hardly, or not at all, be captured by the traditional growth models, since they cannot incorporate the role of the qualitatively new factors of production and products. They consider growth of only a few factors, do not take into consideration more than two or three factors of production (labour, capital and sometimes land or some other natural resources) which are necessarily highly aggregated and therefore, in practice, evaluated only quantitatively, and measure the outcome of production with a single scale (generally with the GDP).

Giving up the explicit consideration of qualitative diversity, these models tacitly assume that it is possible to properly measure and aggregate different types of work, equipment and product. Even when these models address changes in the set of efficient activities<sup>3</sup> at all, it is done in such a manner by that as a result of the newer and newer activities (which were unknown or infeasible before), more and more output can be produced with the same combination of homogeneous labour and capital.

Assuming therefore that the type of the production function of the economic unit in question does not alter, all possible changes in the parameters of the production function are generally considered - after Solow [1957] - technical progress, which result in the gradual movement of the production surface away from the origin, and simultaneously in the approach of the isoquants to the origin. In these models the effects of technical progress are intended to be captured by the *t* time-variable treated as an "explanatory" variable,

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<sup>3</sup> Strictly speaking, those activities which are feasible and efficient in the Pareto sense.

and each factor is incorporated into the category of technical progress, that takes part in the determination of production capacity, but is either unknown, or, for certain reasons, is not taken into consideration explicitly, as an independent variable in the production function

$$Q_t = F(L_t, K_t, t)$$

where  $Q_t$ ,  $L_t$ ,  $K_t$  represent the amount of output, labour input and capital input, respectively.

Thus - as *Gapinski [1974]* remarks -, if each variable which has some effect on the capacity output at some combination of inputs were known, observable and measurable, we could not speak of technical progress, at least not in the above sense.

Technical progress defined in this way can be categorised in different ways. It can be, for example, neutral or non-neutral, depending on whether it changes the value of a given economic variable while that of another variable does not alter, or not. If not, then technical progress has no effect upon the relationship between these two variables, i.e. it is neutral. Otherwise, it is called non-neutral or biased. Of course, depending on the specific choice of the variables, different kinds of neutrality can be defined. In the case of Hicks-neutral technical progress, for example, the chosen variables are the capital/labour ratio and the marginal rate of technical substitution; in the case of Harrod-neutrality, the average- and the marginal productivity of capital; while in the case of Solow-neutrality, the average- and marginal productivity of labour.

According to another distinction, now of greater interest and importance for us, technical progress can be considered to be embodied or disembodied in capital, depending on the way the new inventions resulting from such technical progress can be introduced and utilised in the process of production. If in practice technical progress can be realised only by installation of equipment of the latest type, then it is called *embodied in capital* (or just *embodied* in brief), otherwise it is called *disembodied*.

Traditional growth models based upon aggregate production functions always assume that technical progress is disembodied, i.e.

that the diffusion and general utilisation of the latest inventions do not require any additional investment, but they affect the whole stock of capital in use, immediately. Thus, new technologies are supposed to spread at infinite speed. In contrast, technical progress embodied in capital is not automatic, it improves the efficiency only of the latest equipment, and/or increases the productivity of those who work it. The rate of its diffusion is hence a function of the gross fixed capital formation.

It is to be remarked that embodied technical progress could be related not only to capital, but to labour as well. If it is assumed that technical progress is embodied exclusively in the capital input, then labour is tacitly considered homogeneous in regard to qualification and skill. In reality, however, it is certainly not so, and although it is less obvious than in the case of capital, it is still feasible that succeeding generations of the labour force are able to operate the same equipment with greater and greater productivity, i.e. that technical progress is embodied in labour as well. However, for simplicity, in the following sections of the paper this possibility is ignored and embodied technical progress is assumed to be embodied in capital only.

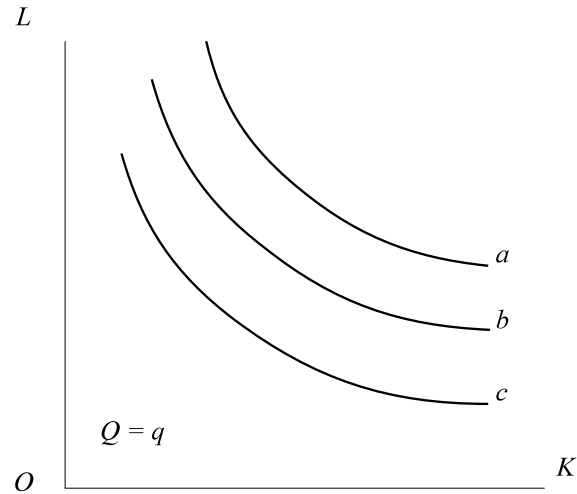
Note also that both the rate and the direction of the hypothetical technical progress can be either exogenous or endogenous elements of a growth model. This distinction depends on whether they are taken as direct functions (usually constant functions) of the  $t$  time variable, or as functions of some economic variable(s)<sup>4</sup>. However, whether technical progress is exogenous or endogenous is not centre to the vintage theory. Thus again, for simplicity, technical progress will be considered an exogenous factor of growth in this paper.

Concerning the set of efficient activities in traditional growth models, capital is always dealt with as a homogeneous factor of production. In these models, there is always a unique set of the efficient activities for the new equipment being installed in a current, or potential investment project, as well as for equipment already in use.

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<sup>4</sup> About endogenous technical change in general see e.g. *Grossman-Helpman [1991]* or *Sato-Suzawa [1983]*, while in relation to vintage models see e.g. *Bosworth [1976]*, *Klundert-Schaik [1978]* and *Gelauff et al. [1985]*.

In consequence of disembodied technical progress the elements of this set can, of course, change from year to year. Assuming that this set can be described with a neo-classical production function (with that of a CES type for example), this process is illustrated in *Figure 1*.



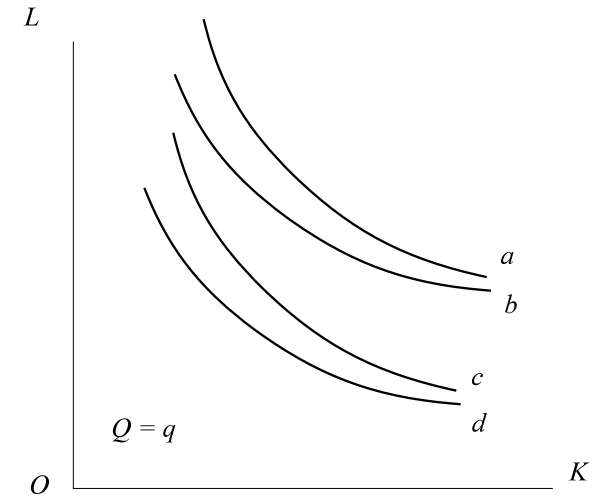
$t' < t'' < t'''$   
 $a: L(t')$  ;  $b: L(t'')$  ;  $c: L(t''')$   
 $L(t)$ : Isoquant for  $q$  units of output in year  $t$

*Figure 1: Isoquants of an Aggregate Production Function in a Traditional Model*

As to this assumption of the traditional growth models, on the basis of practical experience and theoretical considerations, it is reasonable to make a distinction between the set of *ex ante* efficient activities on the one hand, and the set of *ex post* efficient activities on the other hand, and therefore between the production functions which describe these sets, as well.<sup>5</sup> One possible reason of this is

<sup>5</sup> See eg. *Johansen [1959]*.

the fact that - presumably due to some embodied technical progress - the machineries being put into operation by current investments are generally more up-to-date than those which were installed last year.<sup>6</sup>



$t' < t'' < t'''$   
 $a: L(t'; \text{ex ante}) = L(t', t'; \text{ex post})$ ;  $b: L(t', t''; \text{ex post})$ ;  
 $c: L(t''; \text{ex ante}) = L(t'', t''; \text{ex post})$ ;  $d: L(t'', t'''; \text{ex post})$   
 $L(u; \text{ex ante})$ : Isoquant of the *ex ante* production function in year  $u$   
 $L(u, v; \text{ex post})$ : Isoquant of the *ex post* production function (of pieces of equipment installed in year  $u$ ) in year  $v$  ( $v \geq u$ )

*Figure 2. Isoquants of the Ex Ante and Ex Post Production Functions under Embodied Technical Progress*

<sup>6</sup> Another possible reason will be discussed in the next section.

Disembodied technical progress alters both the *ex ante* and *ex post* production functions. In contrast, embodied technical progress influences only the set of *ex ante* efficient activities, that is it affects the technical standard of the investment projects just being started or still planned, but not that of equipment already operating. Therefore, when technical progress is partly disembodied and partly embodied, the sequence of the *ex ante* and *ex post* isoquants may look like in *Figure 2*.

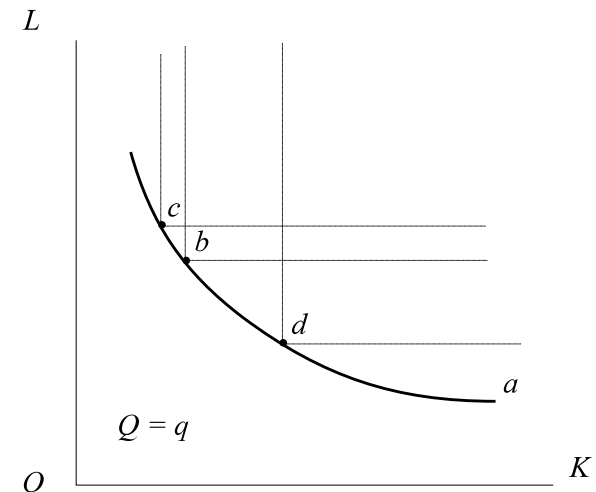
### 2. Ex Ante and Ex Post Substitutabilities between Factors of Production

*Ex ante* factor substitution possibilities imply that one has the choice of several *ex ante* efficient techniques of different capital/labour ratios, while *ex post* factor-substitution possibilities mean that the capital/labour ratios of already completed investment projects can be altered efficiently without any additional cost. Contrary to the general assumption of neo-classical models, in reality *ex post* substitutability is considerably restricted compared to *ex ante* substitutability since designing engineers may choose from the efficient activities certainly with greater freedom than those who are in charge of production. And if the *ex ante* and *ex post* factor substitution possibilities differ in their extents and/or in their domains (extent refers to the elasticity of substitution, while domain to the set of the possible capital/labour ratios), then the *ex ante* and *ex post* production functions cannot be identical, either.

Let us suppose, for example, just for simplicity, that in the economy under consideration in the  $[ \underline{t}, \bar{t} ]$  time period there is no technical change at all, and that the set of the *ex ante* efficient activities - which is therefore unchangeable - can be described by a static CES production function. Moreover assume that *ex post* the production factors are no longer substitutes, which implies that the *ex post* production function is of a Leontief type.

Under these circumstances the *ex post* production functions of the capital units of different investment projects can be the same only if none of the factors and conditions changes in the relevant period which would affect the choice from the set of the *ex ante* efficient activities during planning. But, actually, there is no *a priori* reason to

believe that this condition holds. The situation depicted in *Figure 3* is certainly more realistic.



$t' < t'' < t'''$   
*a*:  $L(\underline{t} \leq t \leq \bar{t}; \text{ex ante})$  ; *b*:  $L(t', t \geq t'; \text{ex post})$  ;  
*c*:  $L(t'', t \geq t''; \text{ex post})$  ; *d*:  $L(t''', t \geq t'''; \text{ex post})$

*Figure 3. Isoquants of the Ex Ante and Ex Post Production Functions under Different Ex Ante and Ex Post Substitutabilities*

### 3. The Concept of Vintage Models

According to the previous two sections, when technical progress is at least partly embodied, and/or the *ex ante* and *ex post* factor substitutabilities are different, capital is a heterogeneous factor of production in respect of the sets of efficient activities. Hence units of the capital stock produced in different years are unlike in their

technical standards, and/or can be run efficiently only under different capital/labour ratios.<sup>7</sup>

*Vintage models* have relevance precisely under these conditions. In these models scilicet, the capital stock is decomposed into subsets according to the year of installation and to the set of *ex post* efficient activities which can be really considered as homogeneous. These subsets are called *vintages*.

Before we proceed three short remarks should be made.

1. From the above definition, it follows that the capital units which belong to the same vintage have the same set of *ex post* efficient activities. But this common set, as well as the *ex post* production function which represents it can alter, however, year by year as a result of disembodied technical progress.<sup>8</sup>

2. This definition does not exclude the possibility of having vintages which differ from each other only in the year of installation or in the set of *ex post* efficient activities.<sup>9</sup> Nevertheless, from now on we ignore this latter possibility, assuming that only one vintage rises each year.

3. The age of a vintage is calculated from the year of installation. Let us assume, that the execution terms of the investment projects - the spans of time from the investment decision till completion - are of the same length. Let us suppose moreover, that in the course of investment decisions entrepreneurs only select *ex ante* efficient activities! Under these circumstances the investment projects, which are set up simultaneously, always result in pieces of capital equipment of the same technical standard.

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<sup>7</sup> Returning now to the example of the previous section it has to be admitted that the stock of capital may be homogeneous (apart from the difference in age), even if the *ex ante* and *ex post* substitutabilities are different, provided there is no embodied technical progress. In this case points *b*, *c* and *d* coincide in *Figure 3*. However, this is only a hypothetical possibility.

<sup>8</sup> This also can be seen in *Figure 4* through 7.

<sup>9</sup> See eg. *Solow [1962]* and *Gomulka [1978]*.

#### 4. Classification of Vintage Models

The sets of *ex ante* and *ex post* efficient activities play key roles in vintage theory. For this reason the most important and specific classification criterion of the vintage models is also related to these sets.

From *ex ante* point of view vintage models are of two different types:

- a) "*clay*-" models, which assume that at all times only one *ex ante* efficient activity exists, or
- b) "*putty*-" models, in which always more than one *ex ante* efficient activities are available simultaneously.

In "*clay*-" models, therefore, the *ex ante* efficient capital/labour ratio is always unique, whereas "*putty*-" models postulate the coexistence of several (in consequence of the combinability of different techniques actually innumerable) *ex ante* efficient capital/labour ratios. From these factor combinations the entrepreneurs may choose the one they prefer at their will.

Vintage models are of two basic types from *ex post* point of view, as well. They can be

- a) "*clay*" models, in which the efficient factor rate of an installed piece of equipment may change only as a result of disembodied technical change, or
- b) "*putty*" models, which assume that there are always more than one *ex post* efficient factor ratios to choose from.

So, while in "*clay*" models - apart from the effect of disembodied technical change - efficient *ex post* factor substitution is not possible, in "*putty*" models factors of production can be directly<sup>10</sup> and efficiently substituted for each other *ex post*, as well.<sup>11</sup>

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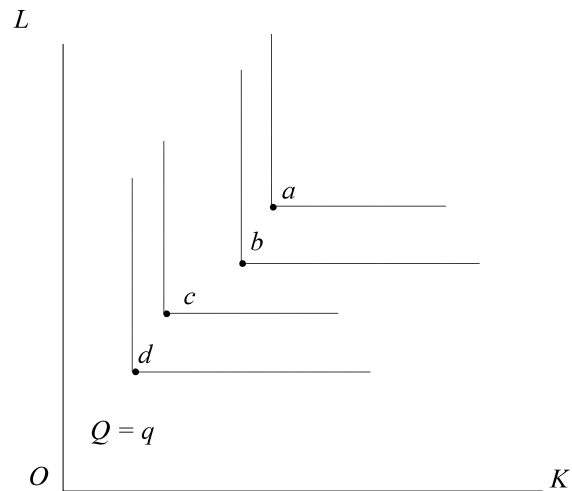
<sup>10</sup> Regarding the whole economy and not only one of its vintages, there is of course always a possible indirect way of efficient factor substitution. Provided that vintages have different efficient *ex post* factor rates, this can be done by installing the latest vintage and scrapping the obsolete, deteriorated ones.

<sup>11</sup> I say "as well", because, if there is no possibility of *ex ante* substitution then one can hardly suppose *ex post* substitutable factors of production.

The union of these two classifications results in three - in theory four, but on the basis of economic interpretability only three - different vintage models:

1. *clay-clay*,
2. *putty-clay* and
3. *putty-putty*.

In the first case labour and capital can be efficiently substituted for each other neither *ex ante*, nor *ex post*. Therefore both the *ex ante* and *ex post* production functions are of Leontief type. Assuming that technical progress is partly embodied and partly disembodied this version is depicted in *Figure 4*.



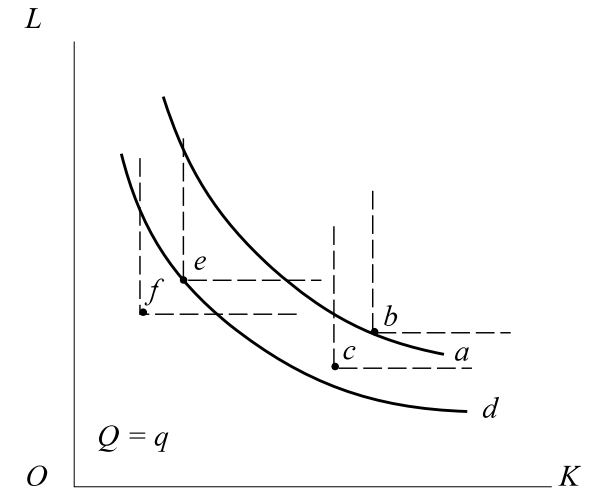
$t' < t'' < t'''$

a:  $L(t'; \text{ex ante}) = L(t', t'; \text{ex post})$ ; b:  $L(t', t''; \text{ex post})$ ;  
c:  $L(t''; \text{ex ante}) = L(t'', t''; \text{ex post})$ ; d:  $L(t'', t''; \text{ex post})$

*Figure 4. Isoquants of a Clay-Clay Economy*

There is no possibility of *ex post* substitution in the second case either, so the *ex post* production function is of a Leontief type again.

But, there are at least two *ex ante* efficient techniques, so the *ex ante* production function can no longer be of a Leontief type. Referring back to *Figure 3* of the previous section, it actually illustrates such a *putty-clay* economy, in which there is no technical change at all. In contrast, in *Figure 5* one can see the isoquants of a *putty-clay* economy with partly embodied and partly disembodied technical progress.



$t' < t'' < t'''$

a:  $L(t'; \text{ex ante})$ ; b:  $L(t', t'; \text{ex post})$ ; c:  $L(t', t''; \text{ex post})$ ;  
d:  $L(t''; \text{ex ante})$ ; e:  $L(t'', t''; \text{ex post})$ ; f:  $L(t'', t''; \text{ex post})$

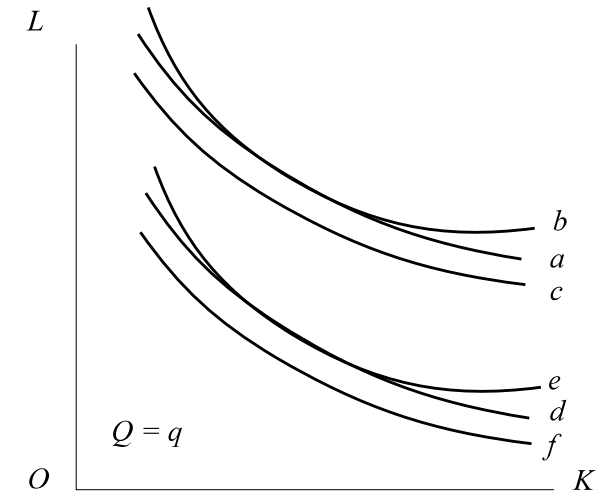
*Figure 5. Isoquants of a Putty-Clay Economy*

Finally, as regards the *putty-putty* models, they postulate that capital and labour are able to substitute for each other efficiently both *ex ante* and *ex post* (and what is more, to the same extent and in the same domain). For this reason in these models neither the *ex ante* nor the *ex post* production functions are of Leontief type. Neo-classical growth models are actually such "degenerate" *putty-putty* models which disregard the possibility of embodied technical change. Therefore in these models vintages differ from each other

only in age, and perhaps in the set of *ex ante* efficient activities. As a matter of fact *Figure 1* of Section 1 illustrates such a case.<sup>12</sup> On the other hand, in *Figure 2* one can see the isoquants of a *putty-putty* economy with partly embodied technical progress.

Most of the vintage models published hitherto (and all of the empirical ones) fall into one of the above-mentioned categories.<sup>13</sup> Besides them there are, however, models which occupy an intermediate position between *putty-clay* and *putty-putty* models. These are the so called *putty-tin*, or *putty-semiclay* models, which although presume that the factor substitution is more restricted *ex post* than *ex ante*, do not deny the possibility of efficient *ex post* substitution.<sup>14</sup> This version is demonstrated in *Figures 6* and *7*. Supposing again that technical progress is embodied and disembodied at the same time, in *Figure 6* the *ex ante* and *ex post* factor substitutabilities differ in their extents, while in *Figure 7* in their domains.

There is hardly any doubt that the *putty-putty* models are the least realistic, while the *putty-tin* models are the most realistic vintage models. However, the *putty-tin* models are the most difficult to specify and to estimate. Therefore, empirical studies mostly focus on *clay-clay* and *putty-clay* models.



$t' < t'' < t'''$

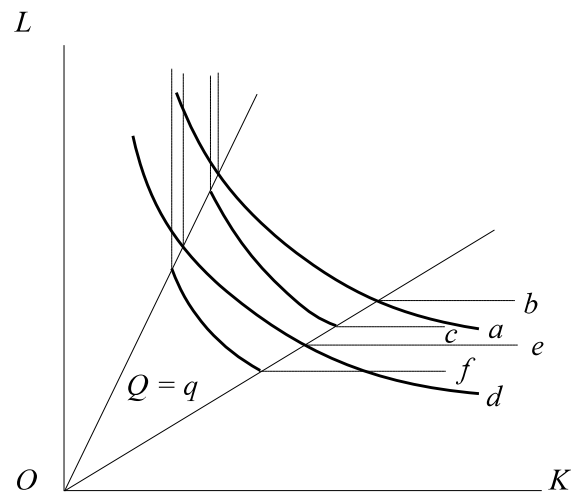
a:  $L(t'; \text{ex ante})$  ; b:  $L(t', t'; \text{ex post})$ ; c:  $L(t', t''; \text{ex post})$ ;  
d:  $L(t''; \text{ex ante})$  ; e:  $L(t'', t''; \text{ex post})$ ; f:  $L(t'', t'''; \text{ex post})$

*Figure 6. Isoquants of a Putty-Tin Economy under Different Extents of Ex Ante and Ex Post Substitutabilities*

<sup>12</sup> From the point of view of the vintage models the curves in *Figure 1* represent the following isoquants: a:  $L(t'; \text{ex ante}) = L(t \leq t \leq t', t'; \text{ex post})$  , b:  $L(t''; \text{ex ante}) = L(t \leq t \leq t'', t''; \text{ex post})$  and finally c:  $L(t''; \text{ex ante}) = L(t \leq t \leq t''', t''; \text{ex post})$ .

<sup>13</sup> Well known examples are: *clay-clay* - Solow et al. [1966] ; *putty-clay* - Johansen [1959] ; *putty-putty* - Solow [1959].

<sup>14</sup> See e.g. Gordon [1973] and Hu [1972].



$t' < t'' < t'''$

a:  $L(t'; \text{ex ante})$ ; b:  $L(t', t'; \text{ex post})$ ; c:  $L(t', t''; \text{ex post})$ ;  
d:  $L(t''; \text{ex ante})$ ; e:  $L(t'', t''; \text{ex post})$ ; f:  $L(t'', t''; \text{ex post})$

*Figure 7. Isoquants of a Putty-Tin Economy under  
Different Domains of Ex Ante and Ex Post  
Substitutabilities*

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