

Equilibrium in a clay-clay or putty-clay economy

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Discussion Paper No. A. 94.13
ISBN 0 85816 983 3
ISSN 1323-2037

September 1994

Abstract:

This paper investigates the conditions for general equilibrium in a *clay-clay* or *putty-clay* economy under at least partly embodied, labour augmenting technical progress. Three segments of the economy are taken into consideration: the money market, the output market, and the labour market. It is shown that although the economy might be thrown out of its balance each year, equilibrium in these markets can be always re-established under certain conditions. The existence of general equilibrium is settled not in the market for goods, but in the labour market.

Equilibrium in a clay-clay or putty-clay economy¹

László Kónya

The first detailed evolving of the vintage theory is owing to *Salter [1960]*. He investigated the effects that embodied technical progress has in a *clay-clay* or *putty-clay* economy² under perfect competition directly on the age of oldest vintage still in use, indirectly (through the lifetime of capital goods) on the price and supply of output, and finally on the equilibrium of the market for goods.

Under certain assumptions he proved that although the output market is thrown out of its balance in consequence of increasing labour productivity each year, its equilibrium is always re-established by new investments which result in increased capacity and decreased price of the output. As regards other segments of the economy - e.g. the money market or the labour market -, Salter left them out of his consideration.

On the other hand, most theoretical vintage models strive to describe such ideal growth paths on which the labour market becomes balanced from time to time, but pay hardly any attention onto the output market.³

The aim of this paper is to show under what common conditions, if at all, can a *clay-clay* or *putty-clay* economy with at least partly embodied⁴, labour augmenting technical progress achieve a general

¹ An earlier version of this paper was published in Hungarian as the second part of *Kónya [1992]*.

² About the concept and types of vintage models see e.g. *Kónya [1994]*.

³ Such as the following e.g.: *Bliss [1968]*, *Gapinski [1981]*, *Phelps [1963]* or *Solow et al. [1966]*.

⁴ Embodied only in capital.

equilibrium, where the output market, the labour market, and also the money market (i.e. savings and investment expenditures) are simultaneously balanced.

The structure of this paper is as follows. Section 1 introduces the basic assumptions. Then Section 2 focuses onto the output market, Section 3 onto the labour market, and the relationship between them is considered in Section 4.

1. Basic Assumptions

Let us consider

A1) an economy under perfect competition,

and suppose that its money market - the less integrant segment of our system - has the following two properties:

A2) the intended saving is a strictly increasing, while

A3) the intended investment is a strictly decreasing continuous function of the interest rate.

Then the actual values of saving, gross investment⁵ and interest rate, respectively, are determined by the equality of intended saving and investment. Moreover, these actual values are equilibrium values at the same time.

Concerning production

A4) let this economy be of a *clay-clay* or *putty-clay* type with

⁵ In consequence of the realised and admitted coexistence of vintages of different ages and technical standards, it is the gross investment (gross capital formation) which gets to be focused in vintage models instead of the net one. The difference of these two amounts to the replacement of that part of the capital stock which is either deteriorated through wear and tear, or becomes obsolete due to technical progress. Since this part is presumed to be replaced by up-to-date equipment, replacement is just as important as net investment in the modernisation of production.

A5) constant return to scale.

This implies, that once a capital equipment has been installed it has rigid factor proportions, i.e. the efficient capital/labour ratio can alter only due to some disembodied technical change. As regards the planning stage, in a *clay-clay* economy there is not *ex ante* factor substitution either, but in a *putty-clay* economy - by definition - always more than one *ex ante* efficient capital/labour ratios exist simultaneously. Let us suppose, however, that

A6) among the *ex ante* efficient factor ratios there is always one, which on the basis of a chosen economic criterion seems superior to the others,

and call the activity of this capital/labour ratio producing one unit of output best-practice technique.

Let us postulate, furthermore, that

A7) the entrepreneurs choose the best-practice technique whenever they decide on a new investment, and that

A8) it takes always G years ($G \geq 0$) to carry out an investment project.

Under these conditions ($Q_{t,t}$) - the capacity output of vintage t^6 in year t - is equal to

$$Q_{t,t} = k_{t-G} I_t \quad (1)$$

where k_{t-G} : capital efficiency of the best-practice technique in year $(t-G)$,

I_t : gross investment put into operation in year t , that is the savings in year $(t-G)$.

Let us assume, moreover, that in this economy

A9) technical progress is at least partly embodied.

⁶ The vintage being installed in year t .

In a *-clay* economy under technical progress exclusively embodied the *ex post* efficient technique of a given vintage remains the same throughout its life, and (because of **A7**) this unique technique is identical with the best-practice technique of the year of the investment decision. On the other hand, if technical progress is only partly embodied, the *ex post* efficient techniques of vintages may alter each year as a result of the disembodied component. But, in both of these cases in consequence of the embodied component of technical progress vintages at disposal differ in their *ex post* efficient techniques.

Contrary to capital

A10) let labour be a homogenous factor of production considering skill, intensity of work and wage

and assume, that making use of homogenous labour and heterogenous capital

A11) homogenous output is produced, which can be used in consumption as well as in production, and is unaffected by technical progress.

Due to perfect competition the products manufactured by different vintages always have identical price, although the average variable costs may vary from vintage to vintage. But, if

A12) all vintages of the period under survey are the same regarding their average uses of material and energy,

then the per unit variable costs may differ only on account of the labour costs, that is because of the difference in necessary labour inputs. Regarding this difference in turn, let us postulate for the sake of simplicity that

A13) the labour productivity of the best-practice technique increases year after year.

Thus, the younger a vintage is in a given year, the less labour it needs for producing one unit of output, and after all - on the basis of **A5**, **A7**, **A10**, **A12** and **A13**, respectively - the less the per unit variable cost of its running is. Consequently, in this economy the

actual labour productivity would be the greatest possible and the per unit variable cost of production the smallest possible, if at all times only the latest vintage were in use. But, because of the durability and cost of capital equipment this case is rather unrealistic.

The diversity in labour productivity of available vintages may be compared with the variety in fertility of land, since

- i) in the short run both of them are given restricting the amount of the greatest possible output,
- ii) the homogenous labour (force) can be employed only with heterogenous labour productivity in either case, and
- iii) the profitable and unprofitable lands and vintages alike are told apart relying on the sign of the possible net return on sale.

For this similarity - following *Marshall [1961]* - we call the net return on sale⁷ of the entrepreneurs owing to the operation of a vintage quasi-rent.

Now, if

A14) we disregard the scrap value of the idle vintages

and assume that

A15) all short-run costs of production are variable costs (costs of labour, material and energy, respectively),

then for the entrepreneurs in a competitive market it is always worth to operate each vintage on which positive quasi-rent can be obtained, since there is no other way of regaining their capital expenditure, at least, in part.

2. Output Market

By means of operating vintage v in year t ($v \leq t$) entrepreneurs might receive quasi-rent equal to

$$R_{v,t} = p_t Q_{v,t} - w_t L_{v,t} - a_t Q_{v,t} \quad (2)$$

where $Q_{v,t}$: capacity output of v vintage v in year t ,
 $L_{v,t}$: capacity demand for labour of vintage v in year t ,
 p_t : price of output unit in year t ,
 w_t : wage rate in year t ,
 a_t : cost of energy and material required to produce one unit of output in year t .

This quantity is non-negative, when

$$p_t \geq \frac{w_t L_{v,t}}{Q_{v,t}} + a_t \quad (3)$$

In consequence of **A13** in this economy consecutive best-practice techniques have higher and higher labour productivity. Thus, if vintage v satisfies this inequality, all younger vintages do meet it, as well.

Rearranging this inequality we obtain an alternative form of the profitability criterion for vintages:

$$\frac{Q_{v,t}}{L_{v,t}} \geq \frac{w_t}{p_t - a_t} = \frac{w_t}{p_t^*} \quad (4)$$

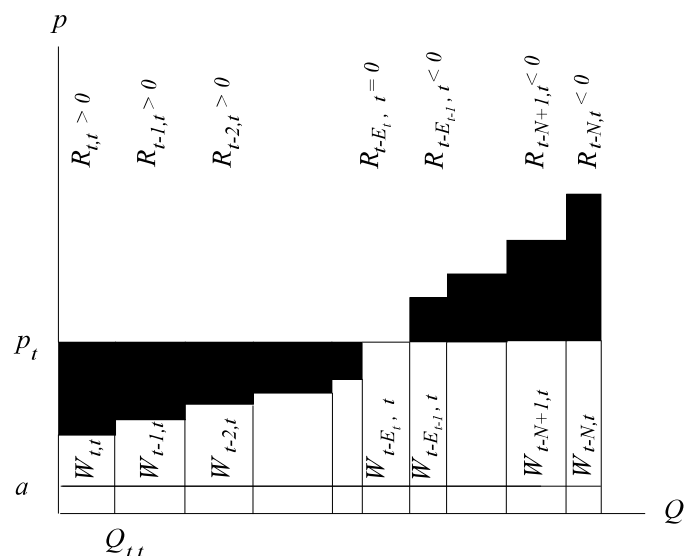
Upon this condition we can uniquely determine the life-span of the oldest vintage, which in year t , under a given real wage rate⁸ may still yield non-negative quasi-rent. This life-span is called the economic lifetime of capital equipment in year t and we denote it by E_t .

This is illustrated in *Figure 1*, where one can see the capacity output (in money terms) of vintages ready to operate in year t . The output of each vintage is divided into costs and quasi-rent, assuming that in

⁷ That is the return on sale after the deduction of costs incurred.

⁸ I use real wage rate in the sense of $w/(p-a)=w/p^*$, that is the ratio of the (nominal) wage rate to the value-added of one unit output.

year t the E_t year-old marginal vintage is a so called *no-rent* vintage, i.e. its operation under the actual real wage rate may result in just zero quasi-rent.



$W_{v,t} = w_t L_{v,t}$: Labour cost of operating vintage v at full capacity in year t

Figure 1. Division of the Output in Money Terms of Vintages into Costs and Quasi-Rents

If we denote the physical lifetime of capital goods by N ($E_t \leq N$), then the total capacity output of this economy in year t can be calculated from

$$Q_t^{(ps)} = \sum_{v=t-N}^t Q_{v,t} \quad (5)$$

while its total capacity demand for labour from

$$L_t^{(pd)} = \sum_{v=t-N}^t L_{v,t} \quad (6)$$

As for the capacity output of vintages which may yield non-negative quasi-rent in year t , it is equal to

$$Q_t^{(s)} = \sum_{v=t-E_t}^t Q_{v,t} \quad (7)$$

and the amount of labour required to operate these vintages is

$$L_t^{(d)} = \sum_{v=t-E_t}^t L_{v,t} \quad (8)$$

- assuming that new vintages are put into operation at the beginning, while worn out and unprofitable vintages are scrapped at the end of each year.

Disregarding disembodied technical change, the total capacity output and the total capacity demand for labour of this economy may change from one year to the other on two accounts: on the one hand due to tear and wear, and on the other hand due to putting the latest vintage into operation. But, the capacity output and demand for labour of vintages of non-negative quasi-rent are affected - through the economic life time of capital equipment - by the price of output and wage rate, as well.

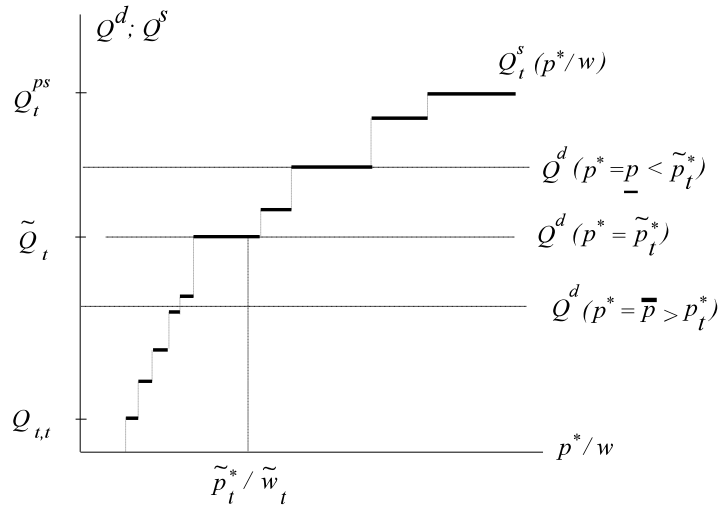
Under perfect competition the supply of output is equal to the capacity output of vintages of non-negative (or only that of strictly positive) quasi-rent, granted that neither the supply of labour, nor the supply of raw materials and energy restrain the operation of these vintages. As one can see in *Figure 1*, this supply - likewise the economic lifetime - is a non-decreasing step-function of the price (p) and a non-increasing step-function of the wage rate (w).

Actually, the supply of the output is determined neither solely by the price, nor solely by the wage rate, but by the ratio of the price to the wage rate through a non-decreasing step-function.⁹ And, if

⁹ This step function is continuous on the right, if the no-rent vintage is in operation, and is continuous on the left, if the no-rent vintage is out of operation.

A16) the demand for the output is affected exclusively by its price (the lower the price is, the higher the demand is),

then the aggregate demand and supply curves of the output can be illustrated in the $(p^*/w; Q)$ plane as in *Figure 2*.



$\bar{p}, \bar{w}, \bar{Q}$: price, wage rate, and output at full-employment.

Figure 2. The Aggregate Demand and Supply Curves of Output

Under the current conditions the demand for and the supply of output may be equal to each other, and thus, the product market may be in equilibrium only at several p values, since the discontinuous supply function and the continuous demand function have common point only in some extreme cases. As you can see in *Figure 2*, there are just as many equilibrium prices as many vintages are at hand. Moreover, if these functions have a common point at all, then they

have not only one, but innumerable.

Let us turn now to the labour market.

3. Labour Market

The aggregate demand function of labour and the aggregate supply function of output are opposite sides of the same question, since both of them can be deduced from the same profitability criterion for vintages - e.g. from relation (3). In turn, as for the supply side, let us assume, that

A17) the supply of labour is independent of both the nominal and real wage rates, and it never exceeds the total capacity demand for labour; i.e.

$$L_t^{(s)} \leq L_t^{(pd)}$$

Then the aggregate demand for and supply of labour can be represented as in *Figure 3*.

If **A17** is not met, then in year t the labour market can not be brought to equilibrium. But, if it holds true, then full-employment may go through in two different ways:

$$a) \quad \sum_{v=t-j+1}^t L_{v,t} < L_t^{(s)} < \sum_{v=t-j}^t L_{v,t} \quad , \quad 1 \leq j \leq N$$

and

$$b) \quad L_t^{(s)} = \sum_{v=t-i+1}^t L_{v,t} \quad , \quad 1 \leq i \leq N$$

respectively.

In the first case the supply of labour in year t exceeds the amount of labour required to operate the j youngest vintages - which according to **A7**, **A9** and **A13** can be run by the highest labour productivity -, but is not sufficient to keep the $(j+1)$ youngest vintages in operation. Consequently, employment can not be full and efficient, unless entrepreneurs (who under perfect competition aim to maximise their profits) keep going each of the j youngest vintages in full, but the

$j+1$ st only in part and the $j+2$ nd already not in the least degree.

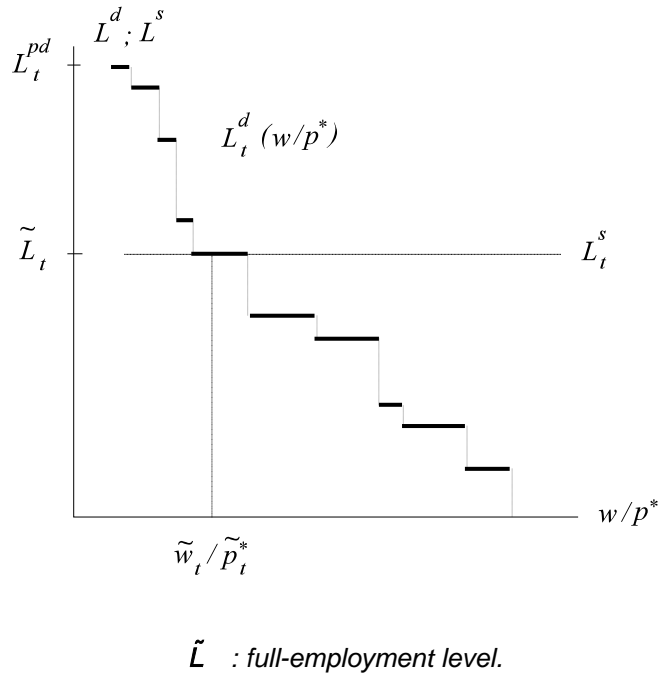


Figure 3. The Aggregate Demand and Supply Curves of Labour

How much is then the real wage rate, which according to **A10** is the same for everybody employed on different vintages?

If labour productivity on the j th vintage were smaller than the real wage rate, that is if

$$\frac{w_t}{p_t^*} > \frac{Q_{t-j+1,t}}{L_{t-j+1,t}}$$

then - according to (4) - this vintage could be run only at a loss, and thus, entrepreneurs would certainly scrap it. But in this case the

demand for labour would not be as high as its supply, and as a consequence, the employees, while competing with each other for jobs, would press the real wage rate downwards.

On the other hand, if

$$\frac{w_t}{p_t^*} < \frac{Q_{t-j+1,t}}{L_{t-j+1,t}}$$

then operating the j th vintage positive quasi-rent might be received. However, owing to the scanty supply of labour this vintage can not be kept going in full. But, as long as the competition for labour does not depress the receivable quasi-rent below zero, entrepreneurs who have the disposal of this marginal vintage do not want to scrap it, whereby press the real wage rate upwards.

Eventually, as a resultant of these two effects employment is full, and the real wage rate is equal to the productivity of labour on the partly operated j th vintage, that is to $Q_{t-j+1,t} / L_{t-j+1,t}$. In turn this means that the quasi-rent which the j th vintage can yield is just zero, or in other words that this marginal vintage is a no-rent vintage.

Moreover, under the current conditions the marginal physical productivity of labour is also equal to the productivity on the no-rent vintage, since if for any reason labour supply increased by one unit, this additional unit would be employed on this marginal vintage, or if labour supply decreased by one unit, it would be withdrawn from this marginal vintage. This implies that the real wage rate and the marginal physical productivity of labour are equal to each other, that is the marginal productivity theory of neo-classical economics remains valid.

In spite of full-employment this state of the labour market can not be considered as equilibrium, unless some of the entrepreneurs who have the disposal of the elements of the no-rent vintage voluntarily cease from their operation.¹⁷

¹⁷ a) In this case the market for goods can also be in equilibrium at the given p_t^* price.

b) If we had started with assuming that the no-rent vintage is never kept in operation, we should now make the following complement: the necessary

Let us turn now to the other possible way of full-employment - indicated above as *b*) - which actually corresponds to the situation depicted *Figure 3*. In this case the supply of labour in year *t* is just equal to the amount of labour required to run the *i* youngest vintages.

If

$$\frac{w_t}{p_t^*} > \frac{Q_{t-i+1,t}}{L_{t-i+1,t}}$$

then the labour market can not get in equilibrium now either, since the *i*th youngest vintage can be run only at a loss. On the other hand, if the real wage rate is greater than labour productivity on the *i-1*st vintage, but not greater than labour productivity on the *i*th vintage - that is, if the real wage rate falls within the

$$\left(\frac{Q_{t-i,t}}{L_{t-i,t}} ; \frac{Q_{t-i+1,t}}{L_{t-i+1,t}} \right]$$

interval, which is open from below but closed from above -, then the supply of and the demand for labour are equal to each other in year *t*.

In this case the marginal physical productivity of labour, similarly to the real wage rate, is not unique either. Since, if labour supply increased by one unit, this additional unit would be employed on the *i+1*st vintage, whereupon total production of year *t* would increase by $Q_{t-i,t} / L_{t-i,t}$ units. On the other hand, if labour supply decreased by one unit, it would not be withdrawn from the *i+1*st vintage, but from the *i*th one, and total production would decrease by $Q_{t-i+1,t} / L_{t-i+1,t}$ units.

Consequently, the marginal physical productivity of labour and the real wage rate would be unique and equal to each other only if $Q_{t-i,t} / L_{t-i,t} = Q_{t-i+1,t} / L_{t-i+1,t}$ were true, that is - contrary to **A13** - the techniques embodied in the *i*th and *i+1*st youngest vintages in year *t* were different only in capital efficiency, at most.

condition of equilibrium is that some of the entrepreneurs be willing to keep the no-rent vintage partly in operation. In any case 'voluntariness' is of primary importance.

These two possible cases of full-employment are therefore entirely different. In case *a*), which is definitely the more realistic, even if employment is full, labour market can be in equilibrium only if some of the entrepreneurs who have the disposal of the no-rent vintage are willing to run it, while the others voluntarily cease from its operation. Nevertheless, real wage rate warranting full-employment is always equal to the marginal physical productivity of labour, that is to the productivity of labour on the no-rent vintage.

Contrarily, in case *b*), though full-employment and the equilibrium of the labour market are interdependent, the equilibrium real wage rate and labour productivity on the marginal vintage are only exceptionally equal to each other.¹⁸

4. General Equilibrium

As we have seen, under the current conditions the market for goods and the labour market have numerous partial equilibrium positions. Nevertheless, although there is a one-to-one relation between production and employment - which is illustrated by the $L_t [Q_t (p_t^* / w_t)]$ curve in *Figure 4* - it is still questionable whether these two markets can be in equilibrium simultaneously, or in other words whether this economy has a general equilibrium situation, or not.¹⁹

To answer this question let us set out from the labour supply - since this is the only variable taken into consideration which depends on time exclusively -, and assume that in year *t* employment is full if the real wage rate is

$$\tilde{w} / \tilde{p}^*$$

This state of the labour market is either of equilibrium or not, but

¹⁸ If labour supply were a strictly increasing function of the real wage rate, we would come to hardly different conclusions. Although the equilibrium real wage rate would be unique, it would not be necessarily equal to the productivity of labour on the marginal vintage.

¹⁹ As regards the existence of general equilibrium, in this instance the money market - which also has a partial equilibrium - is completely irrelevant, since it is supposed to be independent of the two other markets.

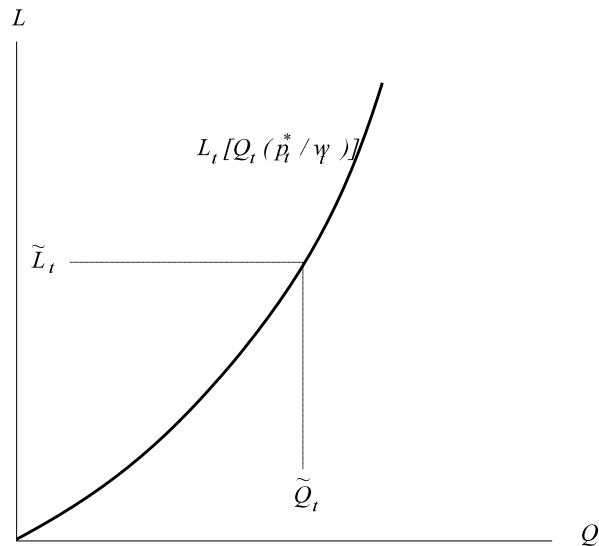


Figure 4. Employment and Production

under this real wage rate general equilibrium may occur obviously only if the demand for and supply of output are equal to each other.

The real wage rate, however has an influence only onto the supply, since the demand for output is supposed to be a continuous, strictly decreasing function of the price of output. And as there is not any special restriction on the domain and range of this function (of course neither the price nor the demand can be negative), if the

$$\lim_{p \rightarrow 0} Q^{(d)}(p) \geq Q_t^{(ps)} \quad ; \quad \lim_{p \rightarrow \infty} Q^{(d)}(p) \leq Q_{t,t}$$

inequalities²⁰ hold, then one can relate to any possible real wage rate

²⁰ If these inequalities are satisfied, then in year t there is certainly such a low output price, under which the demand for output is greater than or equal to the total capacity output of the economy, and on the other hand, there is such a high output price, under which the demand for output is less than or equal to the

one and only one price of the output, under which the market for goods comes to equilibrium.

Thus in any case, when under full-employment and some initial price of the output the demand for, or the supply of the output are in excess, the market for goods can be returned to equilibrium, without affecting the real wage rate and full-employment, simply by changing the price of output and the nominal wage rate in equal proportion into the appropriate direction and in appropriate degree. The new state of the economy developed this way can of course be considered as general equilibrium, only if full-employment means the equilibrium of the labour market at the same time.

Conclusion

At a normal rate of utilisation actual production can never be greater than total capacity output, in fact in most instances it stands underneath this upper bound. Under the current conditions excess capacity may occur for three reasons:

- i) insufficient demand for output,
- ii) insufficient supply of labour and
- iii) too high real wage rate.

From these possible reasons *Salter [1960]* picks up only the first one, although this bottle-neck can be always undone by choosing the appropriate value of the output price. Moreover, as we have seen, the existence of general equilibrium is finally settled not in the market for goods, but in the labour market.

capacity output of the latest vintage.

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